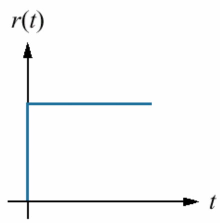
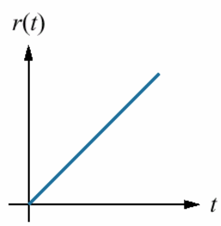
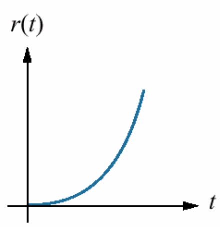
Lecture 7

Steady-State Errors

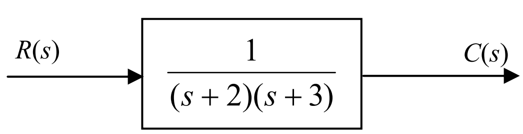
1. Definition and Test Inputs

Steady-state error is the difference between the input and the output for a prescribed test input as $t \to \infty$. The test inputs used for steady-state error analysis and design are summarized in the following table

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

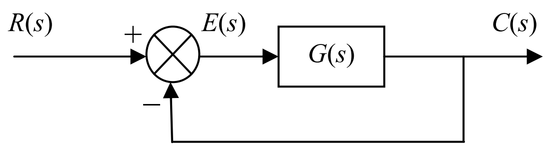
Example

Evaluate the steady-state error for the following system if the system input is a step.



2. Steady-State Error for Unity Feedback System

Consider the following unity feedback system



Recall the closed-loop transfer function of the system, *T*(*s*), is

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

The system output can then be expressed as

$$C(s) = R(s)T(s) = R(s)\frac{G(s)}{1+G(s)}$$

The error, *E*(*s*), between the system input, *R*(*s*), and the system output, *C*(*s*), is

$$E(s) = R(s) - C(s) = R(s) - R(s)T(s) = R(s)(1-T(s))$$

or

$$E(s) = R(s)(1 - \frac{G(s)}{1+G(s)}) = \frac{R(s)}{1+G(s)}$$

Using the final value theorem, the steady-state error is

$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} sR(s)\left(1 - T(s)\right)$$

or

$$e(\infty) = \lim_{s\to 0} \frac{sR(s)}{1+G(s)}$$

Step Input, i.e., $r(t) = u(t)$

Its Laplace transform is $R(s) = \dfrac{1}{s}$.

The steady-state error is $e(\infty) = \lim_{s\to 0} \dfrac{s(1/s)}{1+G(s)} = \dfrac{1}{1+\lim_{s\to 0} G(s)}$.

Ramp input, i.e., $r(t) = tu(t)$

Its Laplace transform is $R(s) = \dfrac{1}{s^2}$.

The steady-state error is $e(\infty) = \lim_{s\to 0} \dfrac{s(1/s^2)}{1+G(s)} = \lim_{s\to 0} \dfrac{1}{s+sG(s)} = \dfrac{1}{\lim_{s\to 0} sG(s)}$.

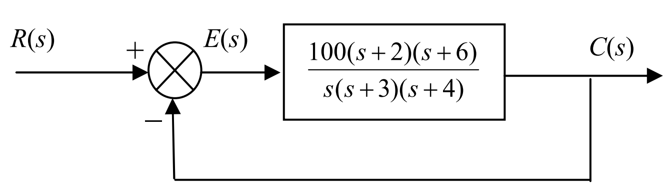
Parabola input, i.e., $r(t) = \dfrac{1}{2}t^2u(t)$

Its Laplace transform is $R(s) = \dfrac{1}{s^3}$.

The steady-state error is $e(\infty) = \lim_{s\to 0} \dfrac{s(1/s^3)}{1+G(s)} = \lim_{s\to 0} \dfrac{1}{s^2+s^2G(s)} = \dfrac{1}{\lim_{s\to 0} s^2G(s)}$.

Example 1

For the unity feedback system shown in the following figure, find the steady-state error if the input is $5tu(t)$.



Example 2

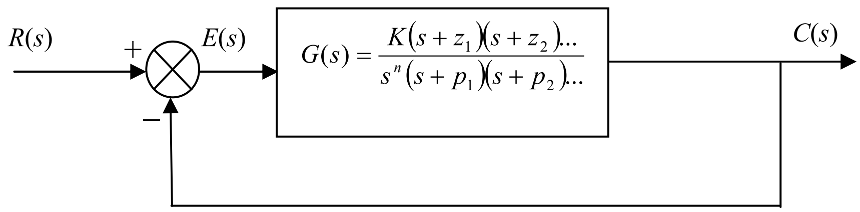
Evaluate the steady-state errors for the unity feedback systems with the open-loop transfer functions of

(1**)** $G(s)=\dfrac{1}{s+2}$, **(2)** $G(s)=\dfrac{1}{s(s+2)}$, **(3)** $G(s)=\dfrac{1}{s^2(s+2)}$

if the system input is a step, a ramp, and a parabola, respectively.

3. System Types

Consider the following unity feedback control system.



The term of s^n in the denominator of $G(s)$ indicates that there are n integrations in the forward path. According to the value of n, systems can be classified into different types. Specifically, a system is called **Type 0 system** if $n = 0$, **Type 1 system** if $n = 1$, or **Type 2** system if $n = 2$, and so on. Note that this classification is different from that of the system order.

Relationship between inputs, system types, and steady-state errors

System / **Input**	Type 0	Type 1	Type 2
Step	Non-zero constant	0	0
Ramp	∞	Non-zero Constant	0
Parabola	∞	∞	Non-zero Constant